Non-asymptotic entanglement distillation

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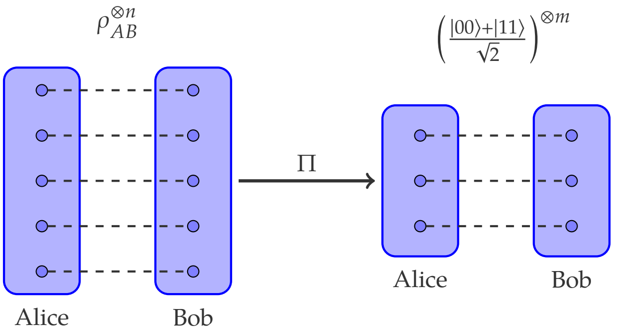
Entanglement distillation [Bennett, DiVincenzo, Smolin, Wootters, 1996]



$$E_D(\rho_{AB}) := \sup\left\{\frac{m}{n} : \lim_{n\to\infty} \underbrace{\inf_{\Pi\in\Omega} \left\|\Pi\left(\rho_{AB}^{\otimes n}\right) - \overset{\frac{|00\rangle+|11\rangle}{\sqrt{2}}}{\overset{\uparrow}{\phi}}{}^{\otimes m}\right\|_1}_{\text{error}} = 0\right\}.$$

Asymptotically, the number of copies of Bell state we can get from per given state ρ.

Entanglement distillation [Bennett, DiVincenzo, Smolin, Wootters, 1996]

$$E_D(\rho_{AB}) := \sup\left\{\frac{m}{n} : \lim_{n\to\infty} \underbrace{\inf_{\Pi\in\Omega}\left\|\Pi\left(\rho_{AB}^{\otimes n}\right) - \phi^{\otimes m}\right\|_1}_{\text{error}} = 0\right\}.$$

$$\phi = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

- Theoretically, fundamental and interesting.
- But not easy to calculate in general.
- From practical point of view, $\lim_{n\to\infty}$ is not possible.

How to do estimation when we only have finite copies of state?

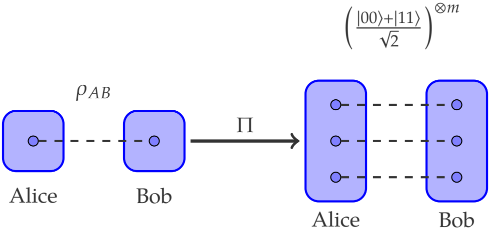
$$\rho_{AB} = 0.7 \cdot |v_1\rangle\langle v_1| + 0.3 \cdot |v_2\rangle\langle v_2|,$$

$$|v_1\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right), \ |v_2\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right).$$

Question:

How many copies of Bell state we can get at most from 222 copies of the state ρ (within the error tolerance 0.01) ?

One-shot entanglement distillation



- Fidelity of distillation [Rains, 2001]:

$$F_\Omega(\rho_{AB}, m) := \max_{\Pi \in \Omega} F\left(\Pi(\rho_{AB}), \phi^{\otimes m}\right), \text{where } \phi = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

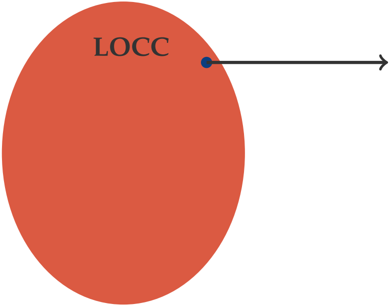
- One-shot distillable entanglement:

$$E^{(1)}_{\Omega,\varepsilon}(\rho_{AB}) := \max\left\{m : 1 - F_\Omega(\rho_{AB}, m) \leq \varepsilon\right\}.$$

- Asymptotic rate:

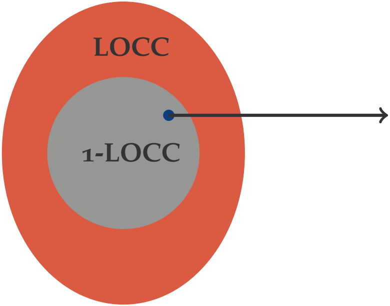
$$E_\Omega(\rho_{AB}) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} \frac{1}{n} E^{(1)}_{\Omega,\varepsilon}\left(\rho_{AB}^{\otimes n}\right).$$

$\Omega \in$ {1-LOCC, LOCC, SEP, PPT}



Local operations and
classical communication $A \longleftrightarrow B$

A hierarchy of operation classes



Local operations and classical communication $A \longrightarrow B$

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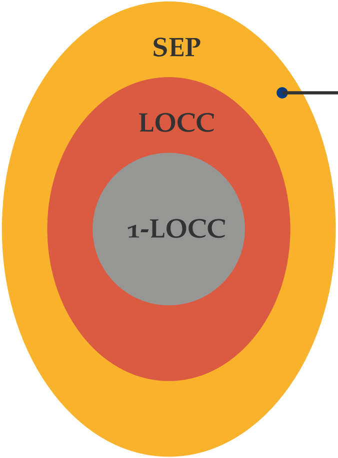
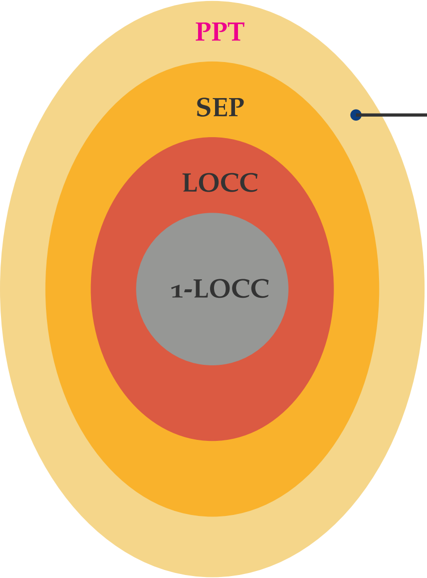


$$J_\Pi = \Pi_{A_1B_1\to A_2B_2}\left(\phi_{A_1B_1:A_1'B_1'}\right)$$

$\rightarrow$ J_Π separable $(A_1'A_2 : B_1'B_2)$

A hierarchy of operation classes



$$J_\Pi = \Pi_{A_1B_1\to A_2B_2}\left(\phi_{A_1B_1:A_1'B_1'}\right)$$

$$J_\Pi^{T_{B_1'B_2}} \geq 0$$

One-shot SDP characterization

For any state ρ_{AB} and error tolerance $\varepsilon \in (0,1)$,

$$E^{(1)}_{PPT,\varepsilon}(\rho_{AB}) = -\log \boxed{\begin{aligned} \min\ & \eta \\ \text{s.t. } & 0 \leq M \leq \mathbb{1}, \\ & \operatorname{Tr} \rho M \geq 1-\varepsilon, \\ & -\eta\mathbb{1} \leq M^{T_B} \leq \eta\mathbb{1}. \end{aligned}}$$

Efficiently computable

Main ingredient of this proof:

Symmetry of maximally entangled state ϕ, i.e., ϕ is invariant under $U \otimes \overline{U}$.

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Efficiently computable

Are we done? How about **large** number of copies $E^{(1)}_{PPT,\varepsilon}(\rho_{AB}^{\otimes n})$?

Quantum hypothesis testing

$$? \quad \rho \in \{\rho_1, \rho_2\}$$

Null: $\rho = \rho_1$ Alternative: $\rho = \rho_2$

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$$\rho \longrightarrow \boxed{\{M_1, M_2\}} \longrightarrow i$$

$i = 1$, accept $\rho = \rho_1$

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$$\rho_1 \longrightarrow \boxed{\{M_1, M_2\}} \longrightarrow 2$$

Type-I error: $\operatorname{Tr} M_2 \rho_1$

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Type-I error: $\operatorname{Tr} M_2 \rho_1$

$$\rho_2 \longrightarrow \boxed{\{M_1, M_2\}} \longrightarrow 1$$

Type-II error: $\operatorname{Tr} M_1 \rho_2$

Quantum hypothesis testing

$$\rho_1 \longrightarrow \boxed{\{M_1, M_2\}} \longrightarrow 2 \qquad \Big| \ \text{Type-I error: } \operatorname{Tr} M_2 \rho_1$$

$$\rho_2 \longrightarrow \boxed{\{M_1, M_2\}} \longrightarrow 1 \qquad \Big| \ \text{Type-II error: } \operatorname{Tr} M_1 \rho_2$$

$$D_H^{\varepsilon}(\rho_1 \| \rho_2) := -\log \boxed{\begin{aligned} \min\ & \operatorname{Tr} M_1 \rho_2 \longrightarrow \text{Type-II error} \\ \text{s.t.}\ & \operatorname{Tr} M_2 \rho_1 \leq \varepsilon, \longrightarrow \text{Type-I error} \\ & M_1, M_2 \geq 0, \\ & M_1 + M_2 = \mathbb{1}. \end{aligned}}$$

Build a connection,

$$\underbrace{E^{(1)}_{PPT,\varepsilon}(\rho_{AB})}_{\text{Distillation}} = \min_{\|G^{T_B}\|_1 \leq 1} \underbrace{D^{\varepsilon}_{H}(\rho_{AB}\|G)}_{\text{Hypothesis testing}}.$$

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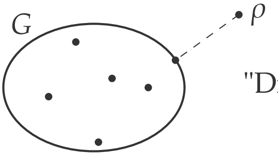

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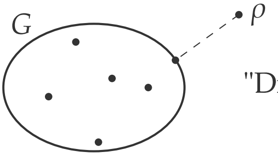

"Distance measure" $\rightarrow D^{\varepsilon}_{H}$

Main ingredient of this proof:

Norm duality between $\|\cdot\|_1$ and $\|\cdot\|_\infty$.

One-shot Hypothesis testing characterization

Build a connection,

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Two Applications:

- Recover the Rains bound.
- Second-order estimation.

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Recover the Rains bound

$$E^{(1)}_{PPT,\varepsilon}(\rho) = \min_{\|G^{T_B}\|_1 \leq 1} D^{\varepsilon}_{H}(\rho\|G).$$

- Rains bound [Rains, 2001; Audenaert, Moor, Vollbrecht, Werner, 2002]

$$R(\rho) = \min_{\sigma\geq 0, \|\sigma^{T_B}\|_1 \leq 1} D(\rho\|\sigma), \quad E_{PPT}(\rho) \leq R(\rho).$$

Recover the Rains bound

$$\boxed{E_{PPT,\varepsilon}^{(1)}(\rho) = \min_{\|G^{T_B}\|_1 \leq 1} D_H^{\varepsilon}(\rho\|G).}$$

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$$\frac{1}{n} E_{PPT,\varepsilon}^{(1)}(\rho^{\otimes n}) = \frac{1}{n} \min_{\|G^{T_{B^n}}\|_1 \leq 1} D_H^{\varepsilon}(\rho^{\otimes n}\|G)$$

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- **Can we improve it by taking other forms of feasible solution?**

Second-order estimation: upper bound

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[Tomamichel & Hayashi, 2013; Li 2014]

$$D^{\varepsilon}_H\left(\rho^{\otimes n}\|\sigma^{\otimes n}\right) = nD(\rho\|\sigma) + \sqrt{nV(\rho\|\sigma)}\,\Phi^{-1}(\varepsilon) + O(\log n).$$

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$$\boxed{E^{(1)}_{PPT,\varepsilon}\left(\rho^{\otimes n}\right) \leq nR(\rho) + \sqrt{nV_R(\rho)}\,\Phi^{-1}(\varepsilon) + O(\log n).}$$

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$$\text{where } V_R(\rho_{AB}) = \begin{cases} \max_{\sigma\in\mathcal{S}_\rho} V(\rho_{AB}\|\sigma_{AB}) & \text{if} \quad 0 < \varepsilon \le 1/2 \\ \min_{\sigma\in\mathcal{S}_\rho} V(\rho_{AB}\|\sigma_{AB}) & \text{if} \quad 1/2 < \varepsilon < 1 \end{cases},$$

and $\mathcal{S}_\rho$ is the set of operators that achieve the minimum of $R(\rho)$

$$D(\rho\|\sigma) := \operatorname{Tr}\rho(\log\rho - \log\sigma), \quad V(\rho\|\sigma) := \operatorname{Tr}\rho(\log\rho - \log\sigma)^2 - D(\rho\|\sigma)^2,$$

Φ^{-1} inverse of the cumulative distribution function of standard normal distribution.

Second-order estimation: lower bound

[Wilde, Tomamichel, Berta, 2016]

$$\boxed{E^{(1)}_{\to,\varepsilon}(\rho_{AB}) \geq -H^{\sqrt{\varepsilon}-\eta}_{\max}(A|B)_\rho + 4\log\eta, \;\text{ where } 0 \leq \eta < \sqrt{\varepsilon}.}$$

1-LOCC

Smooth conditional max-entropy

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$$\boxed{E^{(1)}_{\to,\varepsilon}\left(\rho^{\otimes n}_{AB}\right) \geq nI(A\rangle B)_\rho + \sqrt{nV(A|B)_\rho}\,\Phi^{-1}(\varepsilon) + O(\log n).}$$

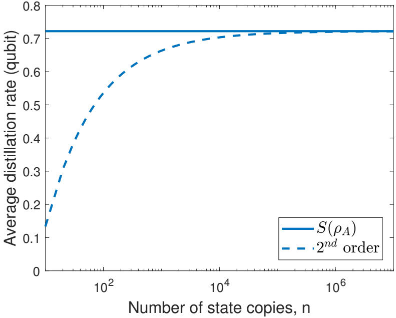
where $\mathrm{I}(\mathrm{A}\rangle\mathrm{B})_\rho := \mathrm{D}\left(\rho_{\mathrm{AB}}\|\mathbb{1}_\mathrm{A}\otimes\rho_\mathrm{B}\right),\ \mathrm{V}(\mathrm{A}|\mathrm{B})_\rho := \mathrm{V}\left(\rho_{\mathrm{AB}}\|\mathbb{1}_\mathrm{A}\otimes\rho_\mathrm{B}\right).$

Examples: pure state

For any pure state ψ, with reduced state $\rho_A = \mathrm{Tr}_B\, \psi$,

$$\boxed{\begin{aligned} E^{(1)}_{\to,\varepsilon}\left(\psi^{\otimes n}\right) = E^{(1)}_{PPT,\varepsilon}\left(\psi^{\otimes n}\right) = nS\left(\rho_A\right) + \\ \sqrt{n\left[\mathrm{Tr}\,\rho_A\left(\log\rho_A\right)^2 - S\left(\rho_A\right)^2\right]}\Phi^{-1}\left(\varepsilon\right) + O\left(\log n\right). \end{aligned}}$$

Remark: Recover [Datta, Leditzky, 2015] 's result about distillable entanglement via LOCC operations for pure states, since 1-LOCC $\subsetneq$ LOCC $\subsetneq$ PPT.



$$\psi = \frac{|00\rangle + 2|11\rangle}{\sqrt{5}}, \ \varepsilon = 0.01.$$

Examples: mixed state

For the state $\rho_{AB} = p|v_1\rangle\langle v_1| + (1-p)\,|v_2\rangle\langle v_2|$, where $p \in (0,1)$,

$$|v_1\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right), \ |v_2\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right),$$

its distillable entanglement is

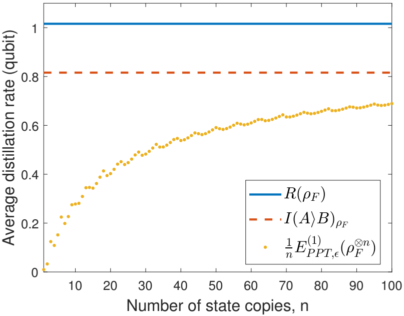
$$\boxed{\begin{aligned} E^{(1)}_{\to,\varepsilon}\left(\rho_{AB}^{\otimes n}\right) = E^{(1)}_{PPT,\varepsilon}\left(\rho_{AB}^{\otimes n}\right) = n\left(1 - h_2(p)\right) + \\ \sqrt{np(1-p)\left(\log\frac{1-p}{p}\right)^2}\,\Phi^{-1}(\varepsilon) + O(\log n). \end{aligned}}$$

where $h_2(p) = -p\log p - (1-p)\log(1-p)$.

Examples: Isotropic state ρ_F $(F = 0.9,\ \varepsilon = 0.001)$

$$\rho_F = (1-F)\frac{\mathbb{1} - \phi(d)}{d^2-1} + F\cdot\phi(d),\ F\in[0,1], \phi(d) = \frac{1}{d}\sum_{i,j=0}^{d-1} |ii\rangle\langle jj|.$$

Small number of copies:



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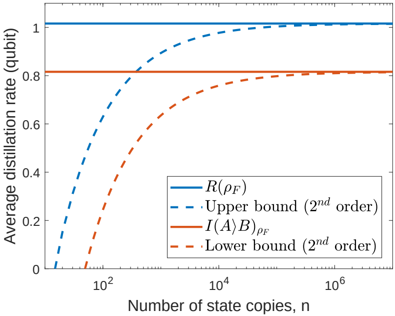
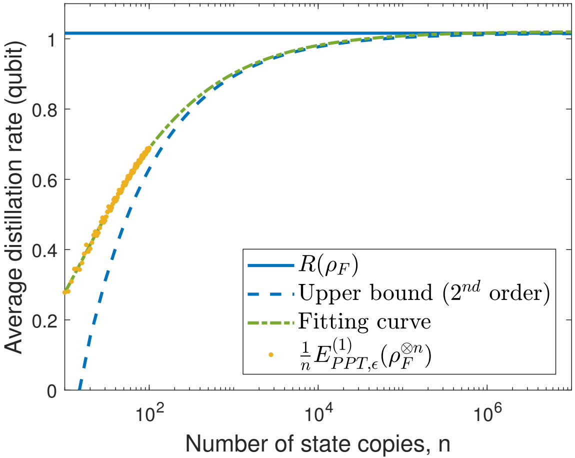


Example: Isotropic state ρ_F ($F = 0.9$, $\varepsilon = 0.001$)



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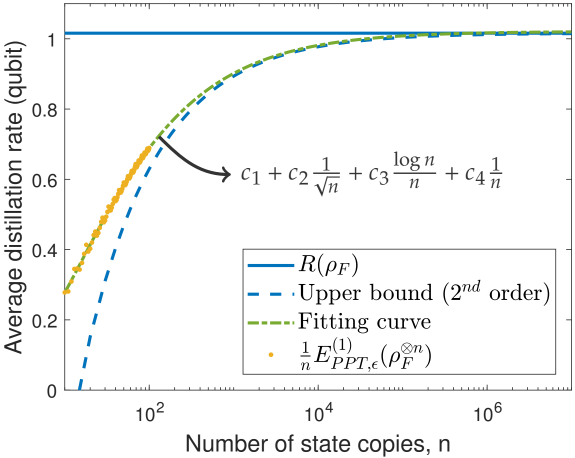

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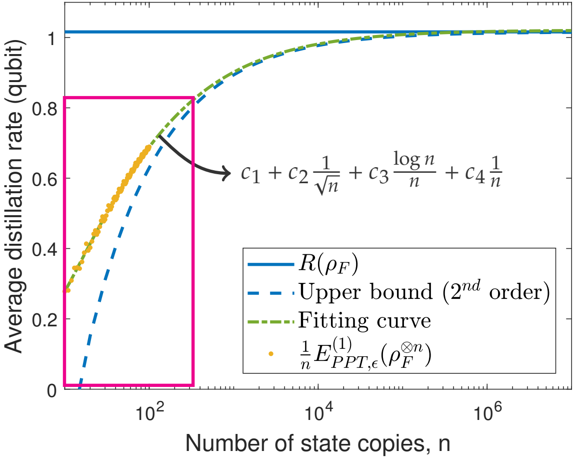

$$\frac{1}{n} E^{(1)}_{PPT,\varepsilon}\left(\rho^{\otimes n}\right) \le R(\rho) + \frac{1}{\sqrt{n}} \sqrt{V_R(\rho)}\, \Phi^{-1}(\varepsilon) + O\left(\frac{\log n}{n}\right).$$

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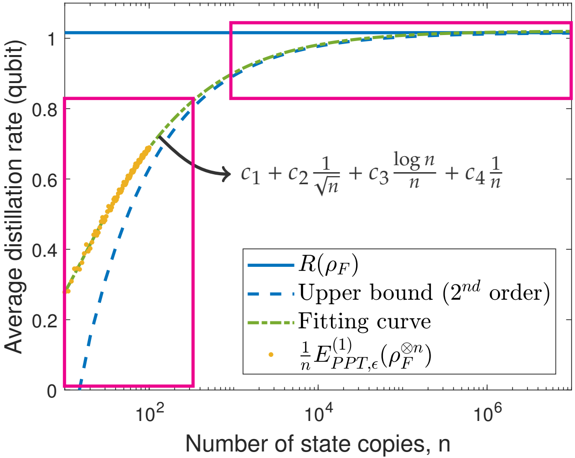

Conjecture: $E_{PPT}(\rho_F) = R(\rho_F)$.

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Summary

$$\boxed{E^{(1)}_{PPT,\varepsilon}(\rho)} \xrightarrow{\text{SDP}} \text{small scale estimation}$$

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$$\downarrow$$

Hypothesis testing

$$E^{(1)}_{PPT,\varepsilon}(\rho) = \min_{\left\|G^{T_B}\right\|_1 \le 1} D^{\varepsilon}_{H}(\rho \| G)$$

Summary

$E^{(1)}_{PPT,\varepsilon}(\rho)$ $\xrightarrow{\text{SDP}}$ small scale estimation

$\downarrow$

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$\swarrow$

Rains bound

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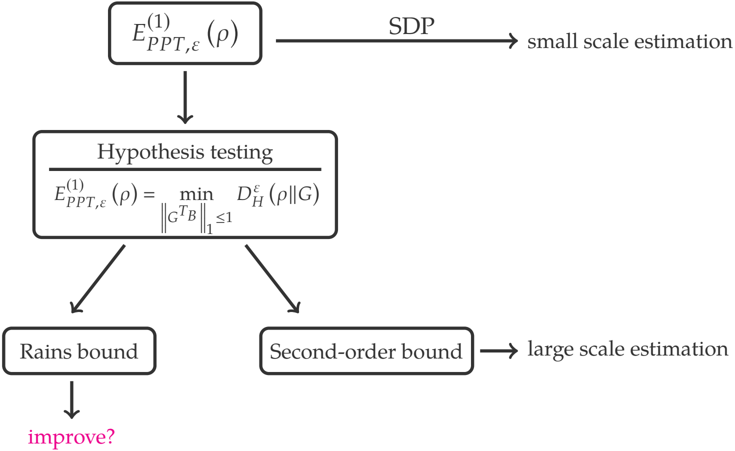
improve?

Summary



$E^{(1)}_{PPT,\varepsilon}(\rho)$
SDP
small scale estimation
Hypothesis testing
$E^{(1)}_{PPT,\varepsilon}(\rho) = \min_{\|G^{T_B}\|_1 \leq 1} D^{\varepsilon}_{H}(\rho\|G)$
Rains bound
Second-order bound
large scale estimation
improve?

Summary

$E^{(1)}_{PPT,\varepsilon}(\rho) \xrightarrow{\text{SDP}}$ small scale estimation

↓

Hypothesis testing

$$E^{(1)}_{PPT,\varepsilon}(\rho) = \min_{\left\|G^{T_B}\right\|_1 \le 1} D_H^{\varepsilon}(\rho \| G)$$

↙ ↘

Rains bound

↓

improve?

Second-order bound $\longrightarrow$ large scale estimation

↓

$E_{PPT}(\rho_F) \stackrel{?}{=} R(\rho_F)$

Summary

$E^{(1)}_{PPT,\varepsilon}(\rho) \xrightarrow{\text{SDP}}$ small scale estimation

$\downarrow$

Hypothesis testing

$$E^{(1)}_{PPT,\varepsilon}(\rho) = \min_{\left\|G^{T_B}\right\|_1 \le 1} D^{\varepsilon}_H(\rho \| G)$$

Entanglement dilution?
Key distillation?
Multi-partite cases?

$\swarrow$ $\searrow$

Rains bound

$\downarrow$

improve?

Second-order bound $\longrightarrow$ large scale estimation

$\downarrow$

$E_{PPT}(\rho_F) \overset{?}{=} R(\rho_F)$

THE END

THANK YOU!

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